KEITH McFARLAND (A)

Removing Pincushion Distortion in a Cathode Ray Tube

"We spent the first three weeks of March 1966 trying to debug the system," said Keith, "but nothing we tried seemed to work. If we got the picture to focus, we couldn't get rid of the 'pincushion'. When we straightened out the 'pincushion', we lost the focus again. Time was really running out on us. Out system had to meet specifications by a certain date or we would have to pay a penalty fee for each day we were late. By that third Friday, I'd decided it was no go. We were up a blind alley. I wanted to try another approach."

Mr. Keith McFarland, an Electrical Engineer in the Link Group of General Precision Corporation was talking about Jet Propulsion Laboratory's (J.P.L.) Video Film Recorder. J.P.L. had subcontracted development of the Recorder (part of the Spacecraft Television Ground Data Handling System) to Link. Keith was assigned as Project Engineer to head the group which designed and constructed the Recorder.

Prepared in the Engineering Casewriting Course, Stanford University, by Charles Fernald under the direction of Karl H. Vesper. Helpful cooperation of Keith McFarland and the Link Group of General Precision Corporation is gratefully acknowledged.

The Video Film Recorder was to take photographs of the TV pictures sent back to earth by such spacecraft as Ranger, Mariner, and Surveyor. The pictures were received as high frequency signals which were demodulated and then projected onto a cathode ray tube (CRT) similar to those used in television sets. One of the specifications set forth by J.P.L. required that the pictures be displayed on a flat faced cathode ray tube, one that was flat to within two thousandths of an inch. This would facilitate taking accurate photographs of the TV pictures, so the photographs could later be enlarged and carefully studied for new information. There was some difficulty however, in producing the pictures on a flat faced CRT. The physics of a CRT employing magnetic deflection are such that there is minimal distortion only if the phosphor coated screen has a spherically curved face. This curved face must have a radius of curvature the same length as the screen's distance from the center of the deflecting field to avoid distortion.

A picture projected on a flat tube face has an inherent distortion known as "pincushion". Near the center of the tube face, there is not much difference in radial distance from the center of the deflecting field to the curved or flat face, and there is little distortion. But as can be seen from Exhibit A-1, the farther from the center of the screen, the greater the difference becomes between the flat and spherical faces. Accordingly, a square, perfectly represented on a curved tube, becomes distorted on a flat one, the greatest distortion occuring at points farthest from the center of the screen, thus:

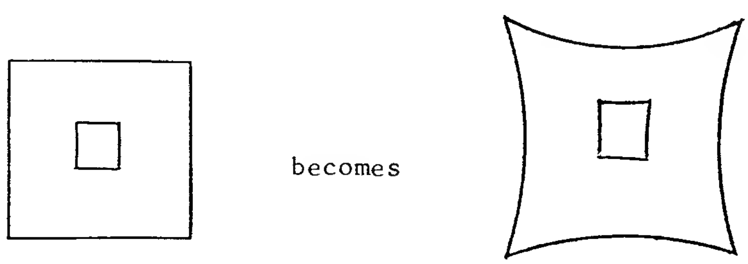


Image on Spherically Curved Tube   Image on Flat Tube

Keith, who had been with Link since graduating in Electrical Engineering from Stanford in 1959, had previously worked with flat faced CRT's in connection with a radar application on a project called a "Land Mass Simulator". However, with the radar a radial sweep picture was obtained and the distortion was in only one dimension, the radial one. (Imagine the second hand sweeping around the face of a clock and leaving shadow images as it passes.) This one-dimensional distortion was eliminated by adding a correction current to the current of the deflection coils in the CRT.

The picture to be transmitted by the J.P.L. spacecraft, however, would involve two dimensions and a radial sweep correction factor would not correct the 'pincushion effect'. Said Keith, "The specifications call for each picture element to be within three tenths of one percent (.3%) of its ideal theoretical position. Let me give you some idea of what this means. Picture elements on a good 21" TV set are around 7% of their ideal theoretical position. J.P.L. requires a picture over twenty times more precise than what you receive at home. With a screen like this, which is flat to within two thousandths of an inch the pincushion distortion can go all the way up to twenty percent at the edges." (Dimensions of the CRT tube to be used appear in Exhibit A-2).

"I first met this 'pincushion' problem when I was working on the Land Mass Simulator. It is a common distortion problem; in fact there are companies which specialize in making corrective devices for it. During the preliminary design, I thought one of these devices would probably bring the distortion within specifications. I went to a catalog and looked up 'pincushion' correctors. A company called CELCO is sort of the Cadillac of the 'pincushion' correctors, so I ordered one of their models that looked like it would do the job."

The CELCO 'pincushion' corrector (see Exhibit A-3) is a magnetic deflection yoke which fits over the neck of the CRT. The yoke produces a static or constant magnetic field which the electron beam passes through after it has been deflected by the horizontal and vertical deflection holes. This additional field causes an extra deflection, the effect of which is enough to straighten out the sides of the picture.

As Project Engineer, Keith's function was to supervise the engineering end of the project. "It's my responsibility to see that the system is working by the contract deadline. If we have trouble with one of the circuits, my job is to work with the engineer until we get the problem ironed out." Keith stated he worked on several other circuit problems of the Video Film Recorder while waiting for the CELCO 'pincushion' corrector to arrive and be installed.

"This project has had its rough spots, but you learn to expect that in R and D contracts. He continued, "That's why R and D contracts are on a cost-plus basis. Things don't always work out the way you plan them to. When you bid on a contract like this, you estimate a cost and a completion date, and you promise a certain degree of technical excellence: 'Our system will have such and such an accuracy, availability, and so on.' Generally it is a trade-off among the three. Sometimes it costs more than you estimated or takes a little longer to deliver; sometimes less. We almost always get the technical excellence we aim for. It's usually the most important.

"We had some typical complications on this job. The specifications were changed a few times. Sometimes this was at our request when we found that the specifications weren't realistic. For example, J.P.L. might specify lenses which just weren't available as off-the-shelf items. They would have had to have been especially developed under another contract with an optical company. Other times we felt some requirements were unnecessary or redundant. When we could show that this was the case, J.P.L. was usually willing to change the specs. J.P.L. initiated some spec changes too. Occasionally they had to request modifications because of design changes in other systems. We got held up a few times by late deliveries from our suppliers and so on.

"Also, there have been several circuit bugs. This 'pincushion' effect has been giving us the most trouble. The CELCO 'pincushion' corrector was installed and tested and we got an unpleasant surprise. The corrector straightened out the sides of the picture pattern within the .3% we wanted, but then we couldn't get the picture to focus properly. The device caused spot growth and increased nonlinearity of the circuit. The result was a fuzzy picture which wouldn't meet the resolution specs. We magnified the picture to see what was happening. It turned out that each picture element

or 'pizel', instead of being a perfect dot, had become cigar shaped. We've spent about three weeks trying to improve the focus and correct for spot growth, but now it's pretty clear that debugging is not the solution. Time is very definitely becoming a problem at this point. Whatever we do has to be done quickly."

Exhibit A-1. Schematic Cross Section of Cathode Ray Tube

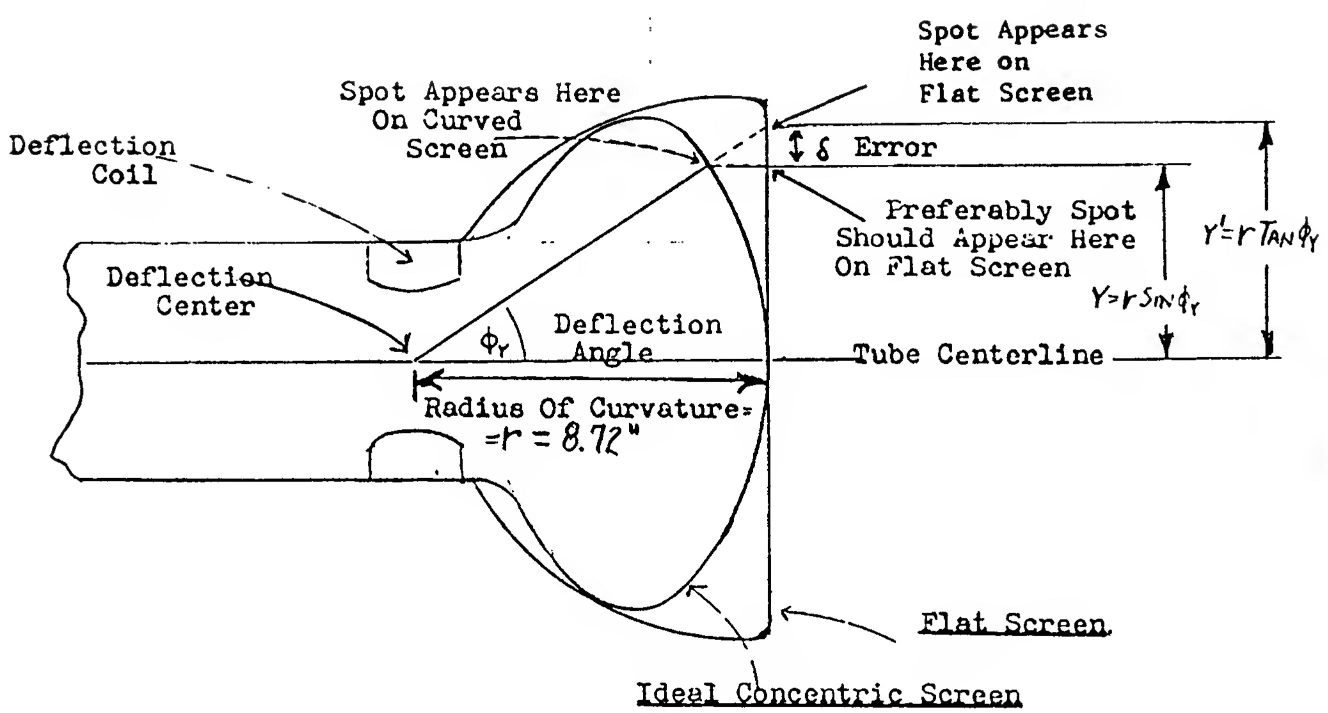
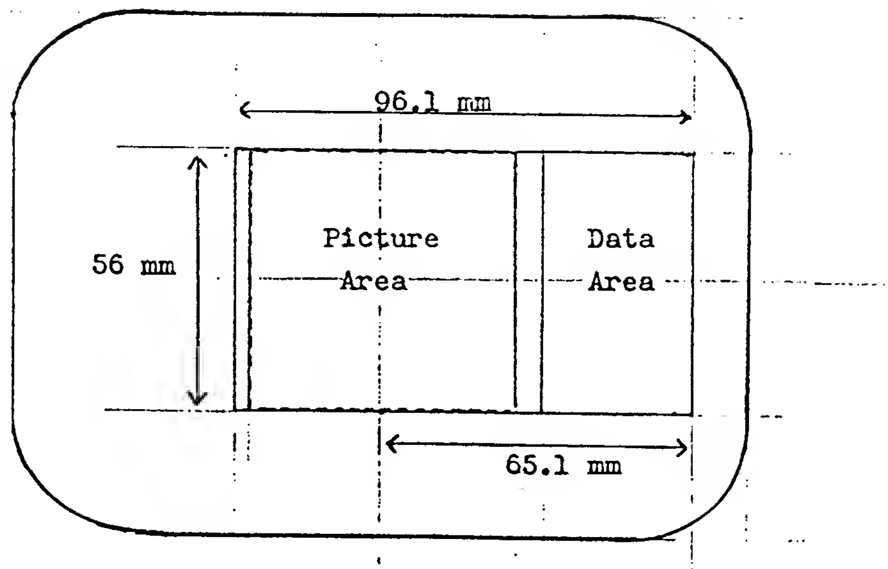
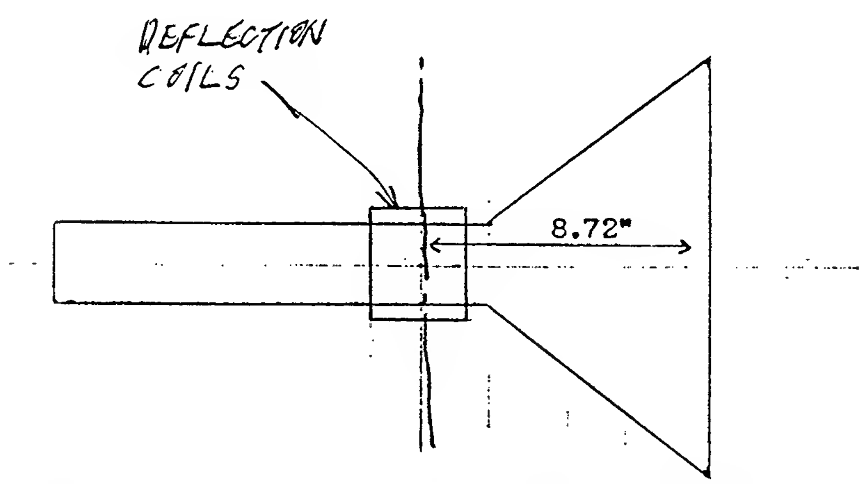


Exhibit A-2 Required Display Dimensions



Cathode Ray Tube Face Showing Actual Display Area

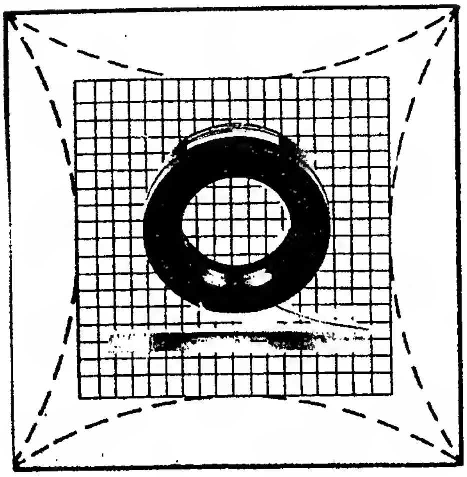


CRT Showing Distance From Center of Deflection to Tube Face

PINCUSHION CORRECTORS TYPEŚ E, U, L, M

PRECISION Celco COMPONENTS

CORRECTIVE FIELDS
FOR
CRT PINCUSHIONING



TYPE E
Papular Precision Pincushian Corrector

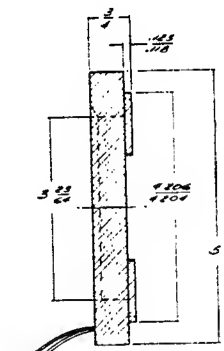
Straight Sides to 0.25%	
Resistance	150 ohms
Current	60 maDC
25 KV with CL1119 CRT	
O.D.	4½"
I.D.	2⁷⁄₁₆"
Thickness	1 ⁵⁄₃₂"
Mounting Bore	$\frac{4.004}{4.002}$ × ½"

CELCO Pincushion Correction Assemblies hove been used by display designers since 1953 far straightening the sides af the well known pincushian pottern on the CRT face.

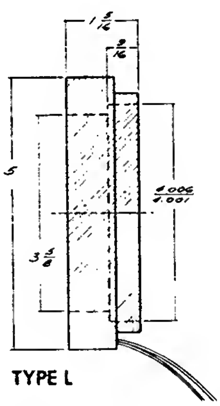
The static correctian field produces magnetic forces that aperate an the electron beam in the drift space between the beam exit af the deflectian yake and the CRT face.

The standord units ore available as shown, or in cambination with ony CELCO Deflectian Yoke ar Deflectron. (See other side af this sheet.)

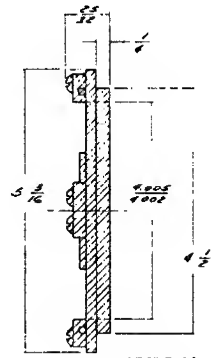
CELCO speciolizes in optimizing field correctars, deflection yokes and CRT's into complete, integrated packoges far minimum spot growth, straightest sides and lineority correction. (See speciol notes on reverse side.)



TYPE U



TYPE L



TYPE M

Ultro-Precision
Straight Sides to 0.1%
Use with CELCO C1628-3 Micro- Positioner
Resistance 100 ohms
Current 125 maDC
10 KV with 5CEP CRT

Low Cast
Straight Sides to 1.0%
Resistance 75 ohms
Current 300 maDC
27 KV with C5A11 CRT

Permanent Magnet
Straight Sides to 1%
No D.C. required
Use with Direct View Displays
on 10UP, 22CP, 5CEP or other CRT's

Standord dato listed obove. Other CRT's ond special applicotian on request. Coll aur Engineering Department.

Celco *Constantine Engineering Laboratories Company*

"For the latest in the science of Electron Beam Control"

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TWX 714 556 9550

SOUTHERN DIVISION
Miami, Florida
Area code: 305 PLaza 1-1132

EASTERN DIVISION
Mahwah, New Jersey
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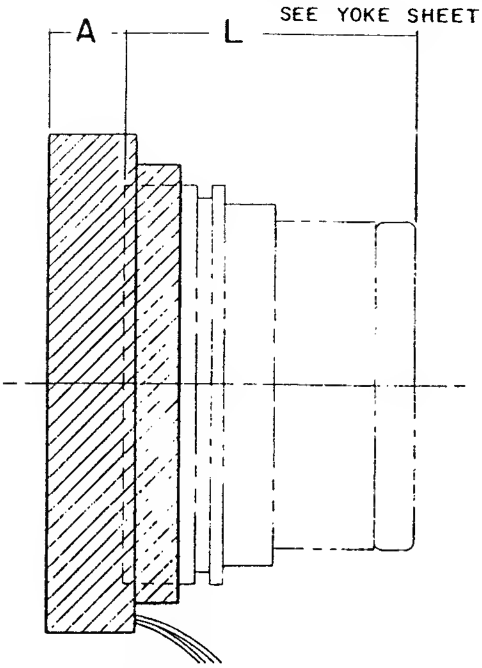
TWX 201 327 1435

STANDARD PINCUSHION CORRECTOR AND DEFLECTION YOKE ASSEMBLIES



OVERALL LENGTHS	
TYPE E	A = 21/32 PLUS L
TYPE L	A = 3/4 PLUS L
TYPE M	A = 35/64 PLUS L

PINCUSHION CORRECTOR AND YOKE

TYPE E—Precision Electromagnetic Pincushion Assembly
TYPE L—Low Cost Electromagnetic Pincushion Assembly
TYPE M—Permanent Magnet Pincushion Assembly
TYPE U—Ultra Precision Electromagnetic Pincushion Assembly

These Pincushion Correctors may be used with any CELCO AY, FY, HY or HD yoke or Deflectron (Ultra High Resolution Yoke) with the housing reversed. The assembly type number becomes AYE521- for a CELCO yoke AY521- (Sheet Y3A) and a Type E pincushion corrector; FYL727- for a CELCO yoke FY727- (Sheet Y17) with a Type L corrector. HDE428- describes a Deflectron HD428- (Sheet D2A) with a Type E corrector. Your CELCO sales engineer can help.

NOTE: Spot growth and linearity are degraded with all ordinary yoke and pincushion corrector assemblies.

SPECIAL NOTE: Celco has developed special Electron-Optical equipment to minimize spot growth in conjunction with pincushion correction.

When the pincushion corrector, the deflection yoke and the cathode ray tube are considered as a unit with all components coupled, a combination may be produced which will meet almost any resolution problem for a display with straight sides.

(Consult our Engineering Department.)

SPECIAL NOTE: Although the pincushion corrector may degrade absolute linearity, straight sides are produced at all values of X-Y current through the deflection yoke. It is possible, therefore, to supply shaped waveforms to the X-Y deflection amplifiers to obtain on-axis correction and to achieve linearity correction for the combined X-Y deflection on the tube face.

These waveforms must be the reverse of those non-linearities which are produced by the CRT face geometry, the yoke and the pincushion corrector.

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KEITH McFARLAND (B)

Keith decided he would try a more analytical look at the pincushion problem. "I went to my bookshelf and thumbed through some of my old text books. In Applied Electronics,[1] I found some of the information on magnetic deflection, but it didn't have anything on the 'pincushion effect,' so I went down the hall to the company library. I picked out a couple of books on CRTs and started reading. You almost always have to brush up on your subject before you can get started. You don't just sit down and start cranking out equations. I worked late that Friday night and took the books home with me over the weekend.

"From my review of the CRT fundamentals, I learned that the 'pincushion effect' is caused by the nonlinear relationship between the angular deflection of the beam, which is proportional to the current in the deflection coils, and the cartesian displacement on the flat face of the tube where the beam hits. The CELCO 'pincushion' corrector was just an extra deflection coil which added fudge factors to the angular deflection of the beam. These fudge factors straightened out the sides of the picture, but they didn't improve the nonlinear relationship between the current in the deflection coils and the cartesian deplacement. In fact, they made it worse. As I said, it got rid of the 'pincushion' but we couldn't get the picture to focus. It seemed as if I might do better if I went to the heart of the problem and tried to modify the currents in the deflection coils. This approach had worked on the Land Mass Simulator and I thought it might work here too. If I could force a linear relationship between the present coil currents and the cartesian deflection by putting something else in the coil circuits, I could get rid of the 'pincushion' and still have the resolution we needed."

Keith drew some sketches to make his point clear. "We have a signal coming from the receiving station that we want to display on our CRT. Now let us assume that we used a spherical face tube." (Illustration follows.)

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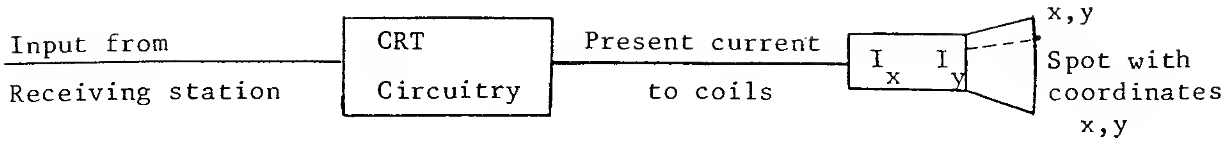
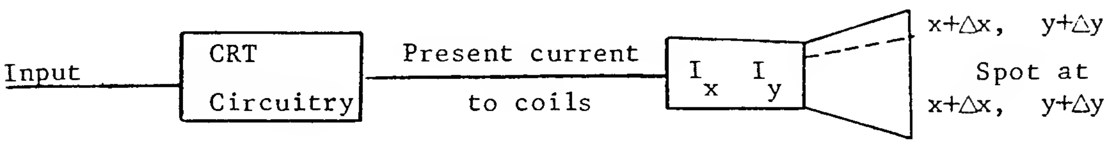
[1] Applied Electronics, Truman S. Gray, 2nd Edition, John Wiley and Sons, New York. 1954. (Text to Keith's introductory course in electrical engineering).

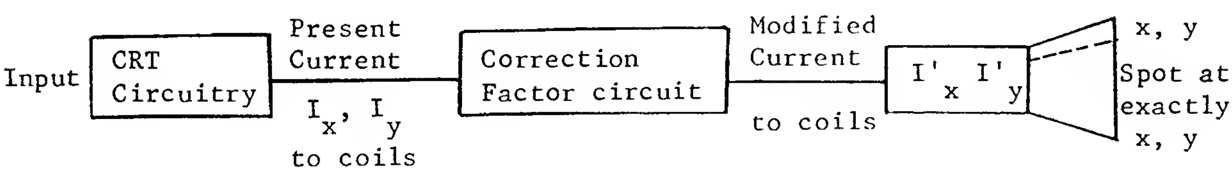


The spot appears where we want it to. The coordinates of a spot on the tube face are linearly proportional to the currents I_x, I_y which go into the deflection coils and we get an undistorted picture. Now, if we use a flat face tube:



The coordinates of the spot are no longer linearly proportional to I_x, I_y, and the spot appears at a distance $\Delta Z = (\Delta x)^2 + (\Delta y)^2$ from where we want it to. Here we have the distorted picture or the 'pincushion effect.'

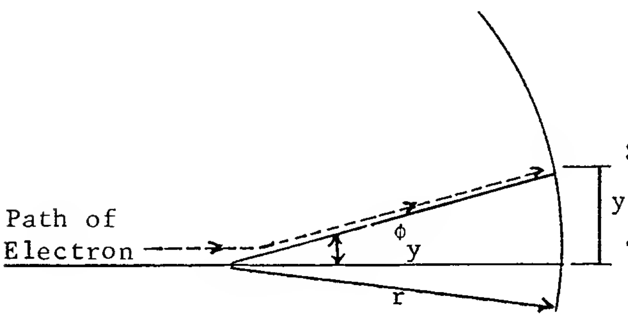
"I think the best way to attack the problem would be to operate on I_x, I_y. This might appear as follows:



Here I'_x, I'_y, are not linearly proportional to x, y, but I_x and I_y, which represent the true picture, are linearly proportional to x, y.

"I still didn't know exactly where I was headed at this point. (Early Saturday morning). I felt that I wanted to correct I_x, I_y, but didn't know just what correction factor I would need. It seemed that if I could express the correction factor mathematically, I would be able to mechanize it. I had worked with mechanization problems before on other projects and I was familiar with some of the techniques. By mechanize I mean design a circuit that takes I_x, I_y, in one end, operates on it and gives I'_x, I'_y, out the other."

Keith said that to find the correction factor he needed, he went back to reviewing magnetic deflection principles for CRTs. "I knew from my reading that the deflection in a spherically faced CRT was proportional to the current in the coils. The relationship was given to me in Applied Electronics as $\sin\phi_y = kI_y$ (where ϕ is the deflection, I is the coil current, and k is a constant). I drew myself a rough sketch to see what this told me."



Since $\sin \phi_y = \frac{y}{r}$, $y = r \sin \phi_y$

Keith said he found that a current I_y in the y deflection coil caused the electron beam to be deflected at an angle ϕ_y. The sine of the angle of deflection was directly proportional to the current in the coil, or $\sin \phi_y = kI_y$, where k is a constant, determined by the characteristics of the tube. From his diagram, he saw that the y position (cartesian coordinate) of the spot was equal to: $y = r(\sin \phi_y)$ or since $\sin \phi_y = kI_y$, $y = rkI_y$. Because of the symmetry of the spherical face, the same relation is true for the x coordinate or $x = rkI_x$. So the deflection was directly proportional to the current.

Then he sketched the case for the flat faced tube to express the deflection coordinates x, y, in terms of the current in the coils, just as he had done for the spherically faced tube. He reasoned as follows: He had a current I_x, I_y, which was supposed to cause a deflection of exactly x, y. Instead, it caused a deflection of $x, y, + \Delta z$. What current then, was necessary to cause a deflection of exactly x, y? This relation had been determined for the spherical case as $y = rkI_y$, $x = rkI_x$.

Keith continued, "The case for the flat face tube was a little harder. But from the geometry and the deflection principles, I finally arrived at

the relation (where θ is the polar coordinate angle or argument in the x, y plane of the tube face): $x = r \sin\theta \tan\Phi$. I knew $\tan\theta$ and Φ in terms of the currents I_x, I_y, in the deflecting coils.

$$\left| \quad \operatorname{Tan}\theta = I_x / I_y, \quad \sin\Phi = k\sqrt{I_x^2 + I_y^2} \quad \right|$$

All I needed now was the relation between $\sin\theta$ and $\tan\theta$ and the relation between $\tan\Phi$ and $\sin\Phi$. I started thumbing through my handbook of math tables and found what I wanted:

$$\tan\Phi = \frac{\sin\Phi}{\sqrt{1 - \sin^2\Phi}} \qquad \sin\theta = \frac{\tan\theta}{\sqrt{1 + \tan^2\theta}}$$

Keith indicated that this gave him:

$$x = \frac{r\tan\theta}{\sqrt{1 + \tan^2\theta}} \qquad \frac{\sin\Phi}{\sqrt{1 - \sin^2\theta}}$$

and substituting $\sin\Phi$ and $\tan\theta$:

$$\tan\theta = \frac{I_x}{I_y} \qquad \sin\Phi = K\sqrt{1^2x + 1^2y}$$

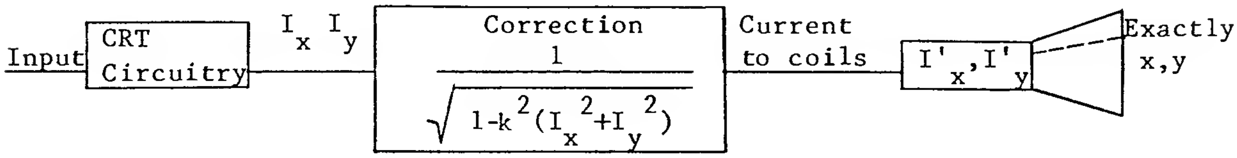
he obtained the relation between the current and the deflection.

$$x = \frac{r\,\frac{I_x}{I_y}\;k\sqrt{I_x^2 + I_y^2}}{\sqrt{\frac{I_x^2}{I_y^2} + 1}\sqrt{1 - k^2(I_x^2 + I_y^2)}} = \frac{rkIx}{\sqrt{1 - k^2(I_x^2 + I_y^2)}}$$

Similarly for the y deflection:

$$y = \frac{rkIy}{\sqrt{1 - k^2(I_x^2 + I_y^2)}}$$

"This was the expression I was looking for," said Keith, "but it surprised me. According to my equation, the x coordinate was a function of both the current in the x deflection coil and the current in the y deflection coil. I knew what correction factor had to go into the little black box, but now I wasn't so sure I could mechanize it."



"From my previous experience with mechanization, I knew we could add factors very accurately but we couldn't multiply factors with an accuracy of more than a few percent. This correction factor called for a multiplication of I_x and I_y by $\frac{1}{\sqrt{1-k^2(I^2_x + I^2_y)}}$. I didn't think we could generate this corrected current accurately enough to bring the picture within the .3% distortion specification.

"Assume for a moment that the correction factor is:

$$\frac{1}{\sqrt{1-k^2(I^2_x + I^2_y)}} = \frac{1}{\sqrt{1-.1}} = \frac{1}{\sqrt{.9}} = \frac{1}{.95}$$

which is a pretty fair approximation. The corrected current is then:

$$I'_x = \frac{I_x}{.95} = 1.05\ I_x$$ If we generate this

multiplication with an accuracy of 3%, then 3% $(1.05\ I_x) \equiv .0315\ I_x$

or 3.15% I_x. So we would still have an error of 3.15% and we would still be far from meeting the .3% error specification."

KEITH McFARLAND (C)

"I had ruled out multiplication as a correction needed to generate an additive correction current." Keith continued, "If we generated the correction current with an accuracy of 3%, then 3% times 5% = .15%. Since we could sum the two currents almost without error, the total error would still only be .15%. Now the problem was to find an additive correction factor which would have the same effect on the current as multiplying by $\frac{1}{\sqrt{1-k^2(I_x^2+I_y^2)}}$

Even approximately the same effect would work because by using addition, there was some extra margin for error. This immediately made me think of infinite series. I knew one could approximate certain functions with the first few terms of an infinite series expansion. In fact, I could approximate these functions as closely as I wanted by just adding more terms. It is one of the old standby methods that electrical engineers use in circuit analysis. I went back to my math handbook and looked up 'binomial series.' I found an expression that looked something like what was needed.

$$(1-bI^2) - \frac{1}{2} = (1 + \frac{b}{2} I^2 + \frac{3}{8} b^2 I^4 + \frac{5}{16} b^3 I^6 + \ldots)$$

"If k^2 were substituted for b and $I_x^2 + I_y^2$ for (I^2), the expression fit perfectly. Now I knew I had the problem licked. All that remained was to decide how many terms of the series were needed. The mechanization would be relatively routine. There were still some loose ends to tie up, but I felt that the worst problems were solved. This took quite a bit of the pressure off.

Keith said that the next morning, Sunday, he began thinking about ways to mechanize the terms of the series he had found. By coincidence it happened that I_x^2, I_y^2, and $(I_x^2 + I_y^2)$ were already being generated for other purposes in the circuits. With the use of log networks, voltage dividers and summing devices, Keith expected he would be able to generate $\sqrt{1-k^2(I_x^2+I_y^2)}$ with sufficient accuracy. The terms of the series, however, required the inverse of this factor, or $\frac{1}{\sqrt{1-k^2(I_x^2+ I_y^2)}}$ instead of $\sqrt{1-k^2(I_x^2+I_y^2)}$.

"I began to wonder if the expression $\dfrac{1}{\sqrt{1 - k^2(I_x^{\,2} + I_y^{\,2})}}$ could be simplified." He said, "The correction factor only had to be generated to about 5% accuracy. The term $\sqrt{1 - k^2(I_x^{\,2} + I_y^{\,2})}$ was available almost as a gift from other parts of the circuit. I wrote down the term $\dfrac{1}{(1 - X^2)}^{1/2}$ and thought about how I might get that denominator up into the numerator. I multiplied top and bottom by $(1 + X^2)^{1/2}$. This gave me:

$$\frac{(1 + X^2)^{1/2}}{(1 + X^2)^{1/2}\ (1 - X^2)^{1/2}} = \frac{(1 + X^2)^{1/2}}{(1 - X^2)^{1/4}}$$

"I knew X^2 would be small so I tried: $(1 - .1)^{1/4} = (.9)^{1/4} - .98 = 1$

Now I had $\dfrac{(1 + X^2)^{1/2}}{\text{approx. } 1}$ which was a much simpler expression. This let me use the binomial expansion:

$$(1 + bI^2)^{1/2} = (1 - \frac{bI^2}{2} + \frac{3}{8} b^2I^4 - \frac{3}{16} b^3I^6 + \ldots - \ldots + \ldots)$$

This would be even easier to mechanize. I wrote out the expression making the necessary substitutions."

$$kI_x[1+k^2(I_x^{\,2}+I_y^{\,2})]^{1/2} = [kI_x - kI_x \frac{k^2}{2}(I_x^{\,2}+I_y^{\,2}) + kI_x \frac{3}{8} k^4(I_x^{\,2}+I_y^{\,2})^2 - \ldots + \ldots - \ldots]$$

"I needed to know how many of these terms I was going to have to work with so I stopped to make some calculations. From one of the sketches I had drawn, I could see the actual deflection coordinates on the face of the CRT were proportional to tan θ rather than sin θ. The error (δ) was simply r (tan θ - sin θ). I wanted to know how many terms of the series I would have to use. Would the first term or the first few terms be enough? If so, what kind of accuracy would they give me?"

The display on the cathode ray tube was designed so that it could be photographed with 70 millimeter film. For this reason, only part of the face of the CRT was used for display. (See Exhibits 1 and 2 in Part A of the case). Of the area of the face that is used, the centermost section of the face was reserved for actual pictures (the smaller the deflection angle and hence the closer to the center of the tube, the more accurate the picture). To the right of the picture or image frame, there was a data frame with numerical and dot coded information. The display area then was not symetric about the center of the tube, and Keith expected separate calculations would have to be made for the maximum error at different edges of the display. The distance from the center of the deflection coils to the tube face along the center line was 8.72".

KEITH McFARLAND (D)

"Before I could determine how many terms of the series were required for .3% accuracy, I needed a better idea of the kind of error I was dealing with. The image area was in millimeters and my tube data was in inches so I made some quick conversions with my slide rule." The conversion factor for millimeters to inches is .0394. The distance from the center of the deflection yoke to the tube face is 8.72". The maximum error on the X axis is on the far edge of the data block.

Maximum X axis deflection = 2.5"

$$\text{Thus } \tan \phi_x = \frac{2.5''}{8.72''} = .2865 \qquad \phi_x = 15.99°$$

$$\sin \phi_x = .2754$$

So max. error on X axis is $r(\tan \phi_x - \sin \phi_x)$

$$8.72(.2865 - .2754) = .097''$$

$$\frac{.097''}{2.5''} = 3.9\%$$

Maximum Y axis deflection = .9"

$$\text{Thus } \tan \phi_y = \frac{.9''}{8.72''} = .1032 \qquad \phi_y = 5.824°$$

$$\sin \phi_y = .10147$$

So max. error on Y axis is $r(\tan \phi_y - \sin \phi_y)$

$$8.72(.1032 - .10147) = .00869''$$

$$\frac{.00869}{.9} = .97\%$$

Maximum diagonal deflection = 2.66"

$$\text{Thus } \tan \phi = \frac{2.66''}{8.72''} = .3042 \qquad \phi = 17.07°$$

$$\sin \phi = .2935$$

So max error on diagonal is $r(\tan \phi - \sin \phi)$

$$8.72(.3042 - .2935) = .117''$$

$$\frac{.117}{2.66} = 4.4\%$$

" I needed a correction factor of at least 4.1%. Since I now had the maximum error for the maximum angle of deflection, and since sin I = kI, I merely plugged the sine of the angle into my series expansion. This gave me:

$$X_o = kI_x(1 - \sin^2/2 + 3/8 \sin^4 - 5/16 \sin^6 + \ldots)$$

or:

$$X_o = kI_x(1 - .043 + .0027 - \ldots)$$

"The first term put me well within the .3% that the specs called for and I knew I could generate it quite accurately. The second term was only about a twentieth of the first and wouldn't have much effect even if I went to the trouble of producing it. So one term was all I needed to get rid of the 'pincushion' and still meet all the resolution specifications.

"By Sunday afternoon, I had a pretty good idea of what the mechanization would be like. I was rather lucky here. I'd done a lot of work with operational amplifiers and log networks before I started on this project. I knew how to use them to get the correction factor I wanted to generate. An engineer with less experience of this sort might have taken a week to do what I was able to do in an afternoon. Of course, he might have found another way to do it in twenty minutes; I don't know. I was just glad that I'd had that experience, particularly because time was getting so short. The last part of the problem was a matter of tying up loose ends. All I had to do was to add the proper scaling factors. By Monday morning I had enough of the details worked out so that my technician could begin a breadboard. By Friday we had a working ciruit."

Keith added that another nice feature of his solution was that the mechanization employed some standard circuits which Link already had in stock. These circuits were:

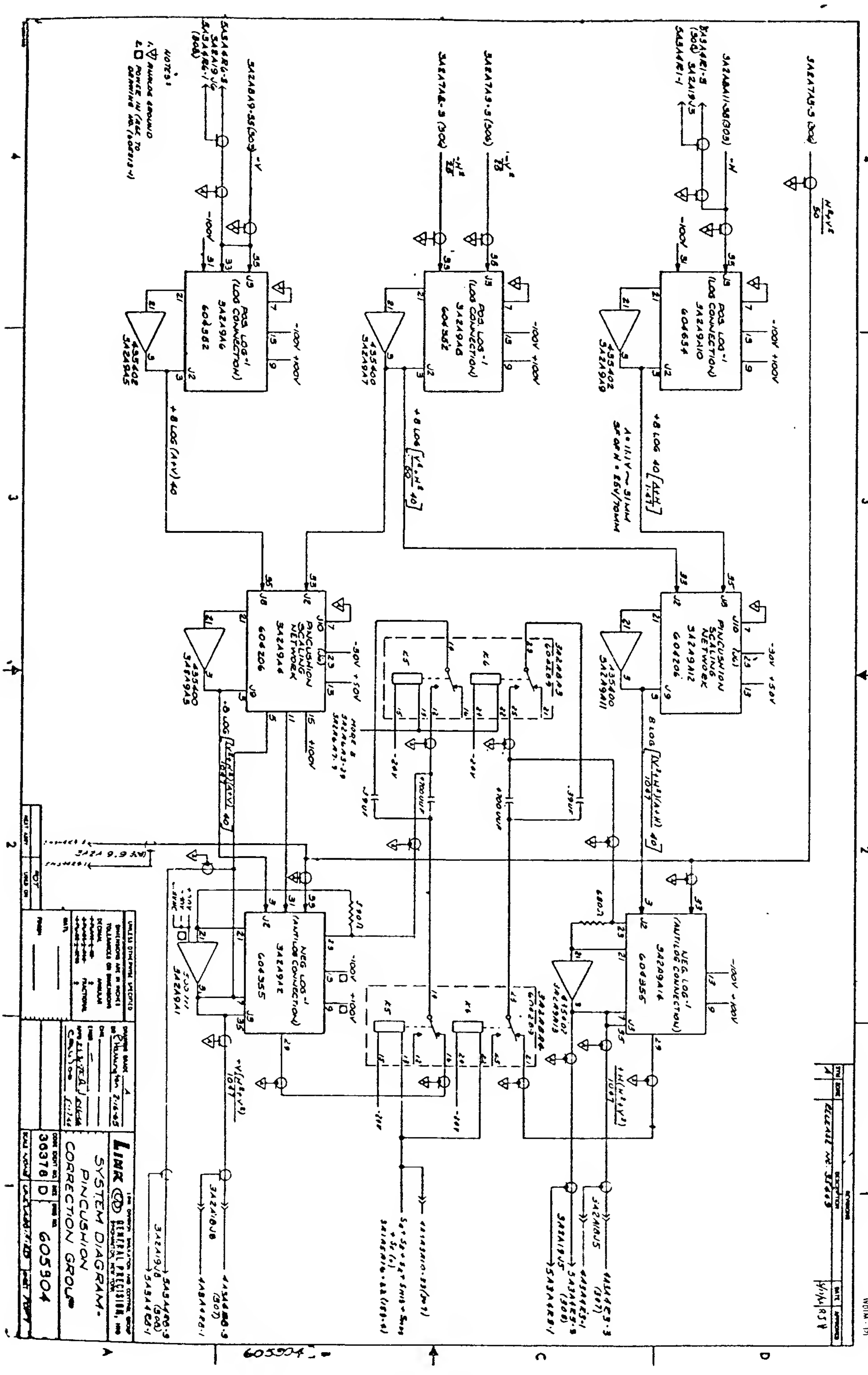
1) Adders, which sum input signals into output.
2) Scaling networks which multiply some input by a constant.
3) Log circuits, which produce an output proportional to the log of the input.
4) Antilog circuits, which are really log circuits used backwards.

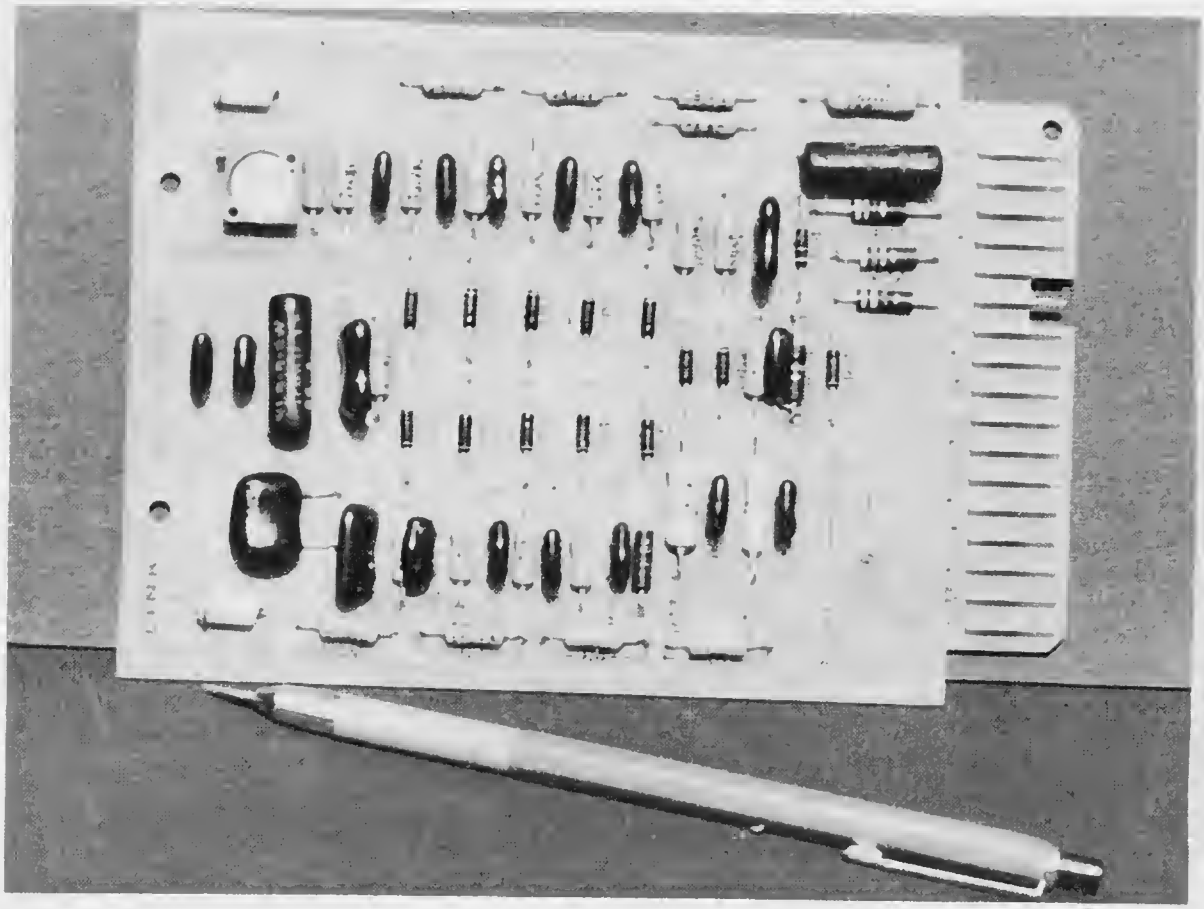
The log circuits enable the engineer to get a product of two factors by summing two or more inputs in an adder and then feeding the adder output into an antilog circuit. It is important to note than an input in any of these circuits becomes inverted (the sign is changed) in the output.

Keith's solution is shown in Exhibit D1. Pictures of the log and summing networks are shown in Exhibit D2. The operational amplifier, symbolized by  is shown in Exhibit D3.

Exhibit D-1 Keith's Solution

ECL 112D





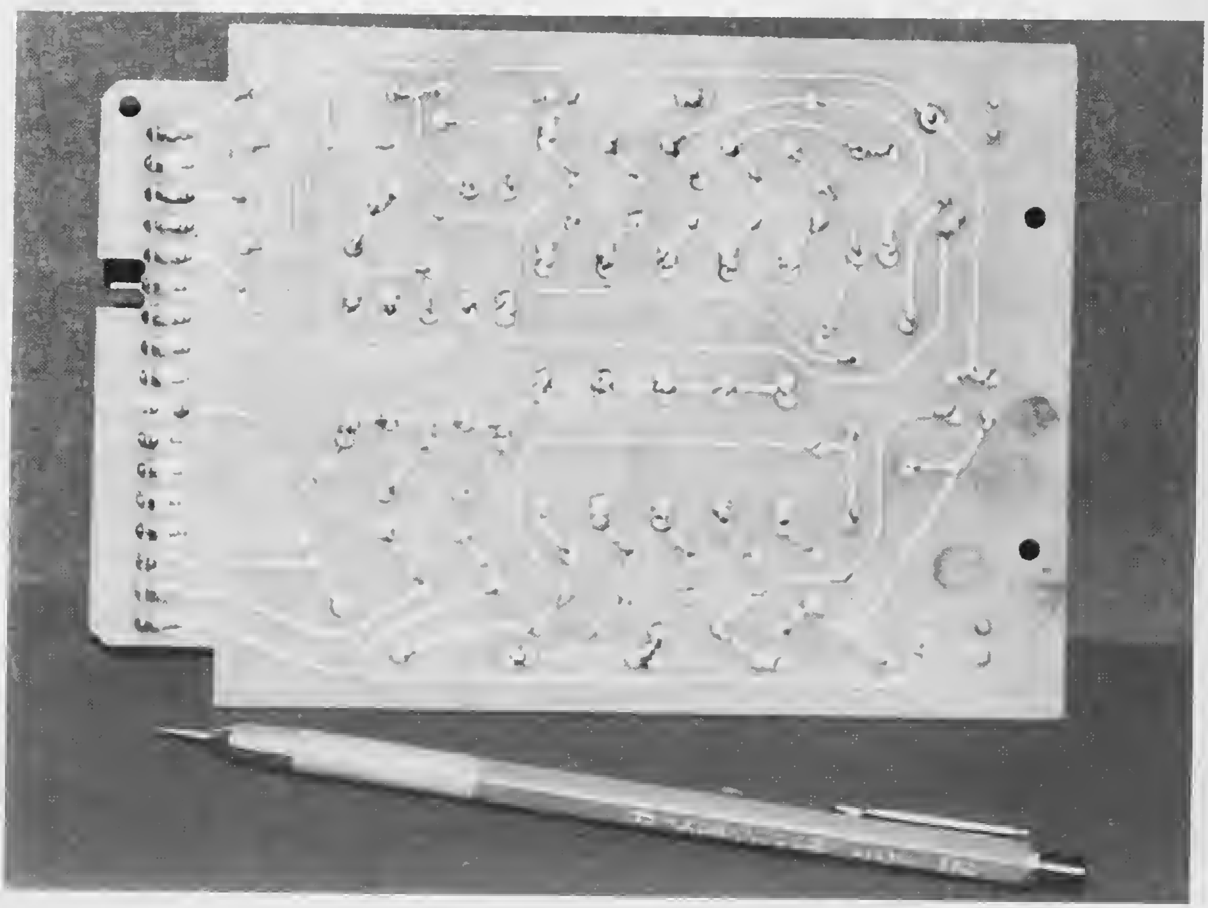


Exhibit D-3. Operational Amplifier